A Supplement to the Tuckerman Tables

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INTRODUCTION

The present work is intended as a supplement to the well known planetary, lunar and solar tables produced by Bryant Tuckerman (1962, 1964). Since these tables appeared- as volumes 56 and 59 of the Memoirs of the Society-they have continued to prove an invaluable aid to historians of astronomy. An important usage is the dating of ancient and medieval astronomical observations but the tables also have wide application in determining the accuracy of early measurements and calculations.

Our supplementary volume owes its origin to the discovery by the authors?F. Richard Stephenson and Michael A. Houlden (1981)? of significant errors in Tuckerman's tabular positions of Mars. Following a query put to one of us (FRS) by the late Professor A.J. Sachs of Brown University, Providence, R.I. regarding the real accuracy of the tables, we made a systematic comparison between Tuckerman's positions for the Sun and planets and those computed from an integrated ephemeris. Only in the case of the longitude of Mars were errors found to be serious but these could amount to as much as 0.7 deg (considerably more than the Moon's apparent diameter). Before outlining the content of the present work, some remarks are necessary on various aspects of Tuckerman's original memoirs.

EXTENT AND PRECISION OF TUCKERMAN'S TABLES

In two remarkably compact volumes, positions of the five bright planets, together with the Moon and Sun, are tabulated over the entire period between 601 BC (or -600) and AD 1649 at intervals of 5 or 10 days. Coordinates are geocentric, relative to the ecliptic-i.e., longitude and latitude. These are more suitable than equatorial co-ordinates (right ascension and declination) for most planetary and lunar calculations and interpolation is much easier. In the case of the Moon and the more rapidly moving planets Mercury and Venus, positions are tabulated every five days, while for the Sun, Mars, Jupiter and Saturn the corresponding interval is 10 days. The hour selected is 16 h UT (4 p.m. Greenwich Civil Time or roughly 7 p.m. at both Babylon and Baghdad).

Tuckerman computed planetary and solar positions to the nearest 0.01 deg, which - provided this accuracy is realised - is more than sufficient for any practical purpose; not until the late seventeenth century was higher precision achieved in measurement. As he stated, it should be possible to interpolate satisfactorily all coordinates except the longitude of Mercury when this planet happened to be near inferior conjunction with the Sun. In the case of the Moon, which typically moves through more than 60 degrees every 5 days, there would have been little justification for tabulating positions to the same accuracy since interpolation would not be possible. Instead, Tuckerman rounded both the lunar longitude and latitude to the nearest 0.1 deg. At this level of precision, interpolation again becomes reasonably practicable. However, contrary to Tuckerman's suggestion, the lunar data are not really adequate for analysing such precise observations as eclipses and occultations. In our opinion, the lunar tables are mainly useful as a guide to the approximate location of the Moon; in any case many lunar calculations require a fairly substantial correction for parallax. Only the planetary and solar data are of value for detailed investigations, but here the applications are extensive. A knowledge of the real accuracy of the tabular co-ordinates is thus of prime importance.

In computing the data for the tables, Tuckerman rather surprisingly adopted what was in general outmoded orbital theory. He used the theory of Leverrier (1858-1861) for the Sun and inner planets, that of Gaillot (1904, 1913) for the outer planets and that of Hansen (1857) for the Moon; in each case he adopted modifications to certain of the orbital elements as derived by Schoch (1926). None of the theories cited had formed the basis of the American Ephemeris since well before 1930. However, despite these cautionary remarks, it is not our purpose to criticise Tuckerman's choice of orbital theory. Our main concern is the accuracy of the data in the published tables themselves.

The question of the true precision of the data tabulated by Tuckerman was first considered in detail by the present authors (Stephenson and Houlden, 1981). For this purpose, the then recently developed Long Export Ephemeris (code name DE 102) was used (Newhall et al., 1983). This ephemeris is based on a systematic numerical integration of the equations of motion of the planets. [ii] The advantage of this type of ephemeris is that a "theory," such as was used to construct Tuckerman's tables, is not needed. Given precise masses and accurate starting positions and velocities for each planet, then in principle an ephemeris can be calculated at any time in the past or future. A typical step proceeds as follows. Starting with some moment at which the rectangular heliocenttic co-ordinates and velocities of the planets are all accurately known, the total force on each planet due to the gravitational action of the Sun and the remaining planets is calculated. The position and velocity of each planet at some neighbouring moment (typically 12 hours away) is then obtained by integrating the equations of motion. The process is then repeated as often as required. DE 102 covers the entire period between 1411 BC (-1410) and AD 3002.

The time system adopted for DE 102 is ET (Ephemeris Time), whereas that used by Tuckerman was UT (Universal Time). In order to effect direct comparison between Tuckerman's tabular co-ordinates and the corresponding DE 102 values, we derived the following expression for AT (ET - UT) by comparing Tuckerman's adopted expression for the solar mean longitude with that deduced by Newcomb (1895) which defines ET:

$$\Delta T = +4.87 + 35.06\,T + 36.79\,T^2 \text{ seconds} \quad (1)$$

Here T is in Julian centuries of 36525 days, measured from the epoch 1900.0.

As might be expected, there is excellent accord between the planetary latitudes tabulated by Tuckerman and those calculated from DE 102. The latitudes of the planets are always fairly small; only Venus moves very far from the ecliptic (to a maximum of about 8 deg). However, the tabular longitudes require detailed discussion. For Mercury, Venus and the Sun, we found the agreement between the tabular and calculated longitudes to be very satisfactory. As far back as 601 BC (the epoch at which the tables commence) discrepancies of more than 0.02 deg are very rare and further are essentially random. In a sample of 200 positions for each object, the maximum error was found to be only 0.05 deg (for Venus). On the contrary, the deviations for Mars were disturbingly large, reaching 0.7 deg around 600 BC see Fig 1. Huber (1983) pointed out that the cause of these discrepancies for Mars was Tuckerman's inadvertent choice of incorrect orbital elements for this planet; these corresponded to ET rather than UT. In particular, Huber demonstrated that the replacement of the quadratic term in Tuckerman's adopted expression for the mean longitude of Mars by its equivalent value in UT gave good accord with DE 102 (deviations as small as 0.05 deg).

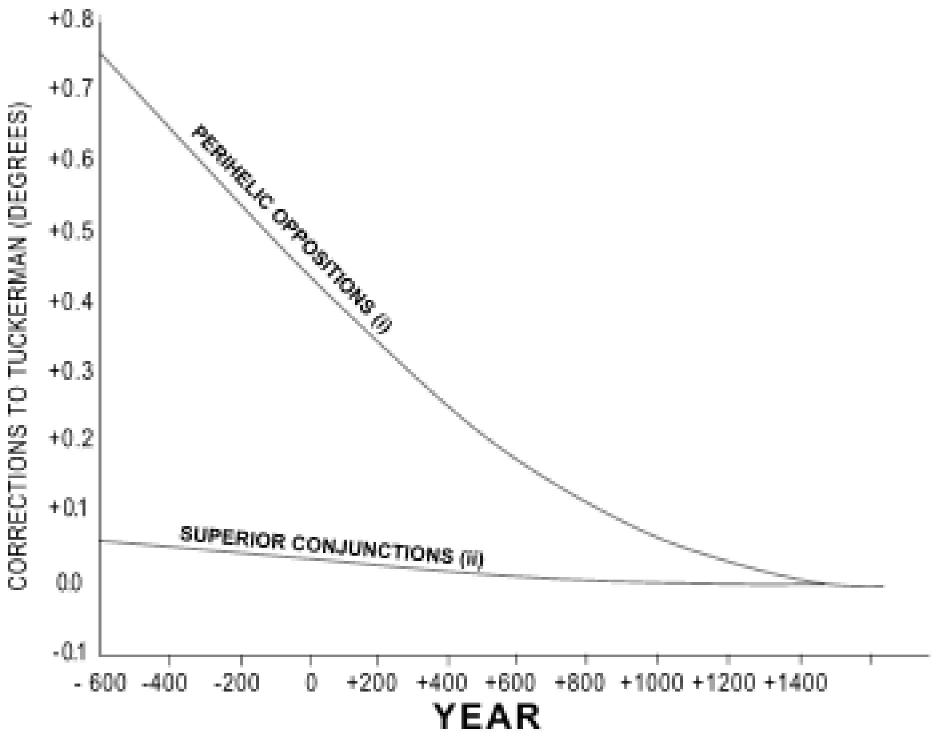

Fig 1. Deviations between Tuckerman's tabular longitudes of Mars and those computed from DE 102 at (i) perihelic oppositions (maximum discrepancy) and (ii) superior conjunctions (minimum discrepancy)

For the outer planets Jupiter and Saturn, the longitudes tabulated by Tuckerman are in very good agreement with the DE 102 data. In the case of Jupiter, over the entire period since about AD 200 the maximum discrepancy is as small as 0.03 deg. Although before that date deviations as large as 0.1 deg may occur, a smooth correction curve can be produced - see Fig 2. Use of this curve enables the tabular longitudes of Jupiter to be reduced to the equivalent DE 102 values with an accuracy of about 0.02 deg as far back as the beginning of the tables. It is evident from Fig 2 that the discrepancies are approximately periodic with increasing amplitude going backwards in time. For Saturn, we have deduced a similar curve (Fig. 3) which allows the tabular longitudes to be corrected with errors as small as 0.03 deg at any time since about 300 BC.

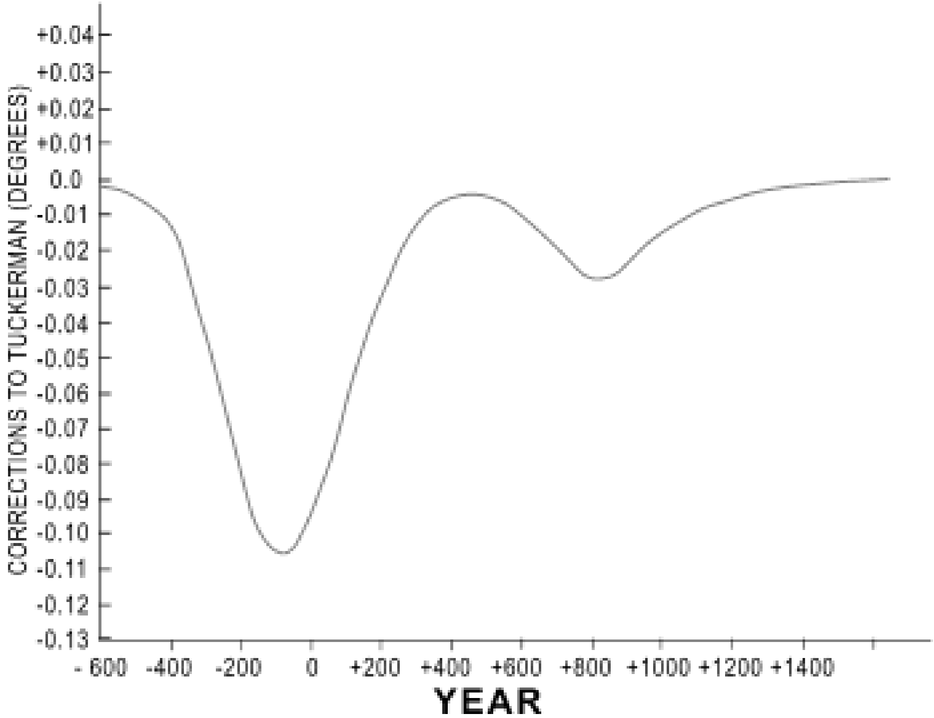

Fig 2. Mean corrections to Tuckerman's tabular longitudes of Jupiter in order to obtain best agreement with longitudes computed from DE 102

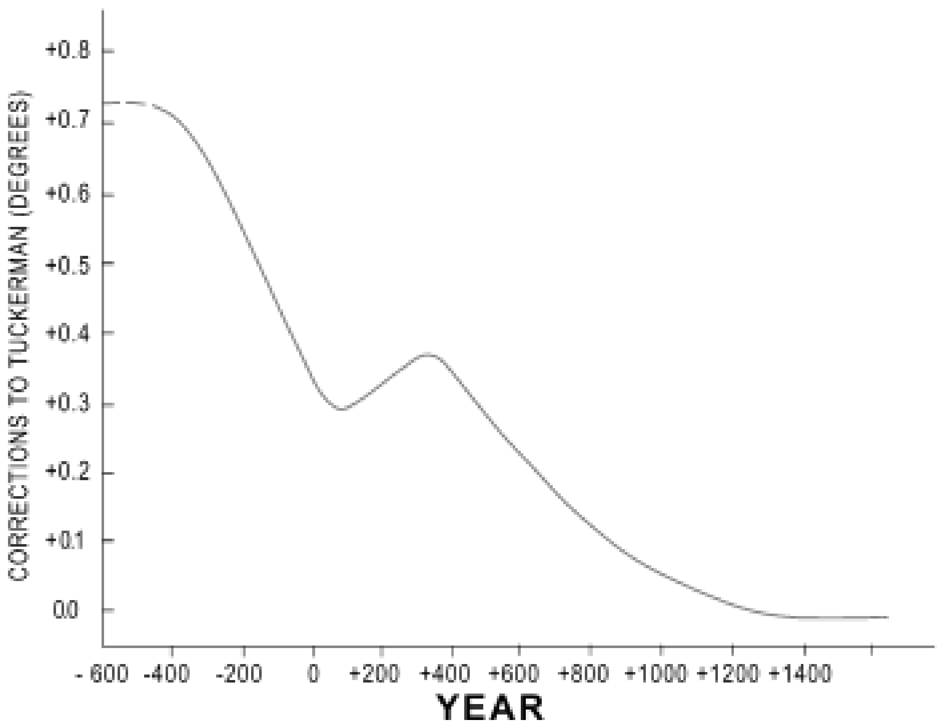

Fig 3. Mean corrections to Tuckerman's tabular longitudes of Saturn in order to obtain best agreement with longitudes computed from DE 102

In earlier centuries the scatter of individual values becomes rather larger-approaching 0. 1 deg - so that at this period only comparatively rough, but still useful estimates of the longitude of Saturn can be made from the tables.

OUTLINE OF THE PRESENT TABLES

In producing the present tables we have two main objectives-to make available revised positions for Mars throughout the period 601 BC to AD 1649 and to enable the apparent magnitude of each planet to be estimated at any time during this period.

Using the integrated ephemeris DE 102, we have computed the longitude and latitude of Mars to the nearest 0.01 deg at 10 day intervals for the same time of day as selected by Tuckerman (16 h UT). As a check on accuracy, we have compared a series of our calculated longitudes for the planet with those based on an orbital theory for Mars which until very recently was used in generating positions for the Astronomical Almanac. This is the theory of Newcomb (1898), with corrections derived by Ross (1917). Co-ordinates of Mars deduced from the Newcomb-Ross theory were kindly supplied by B. Emerson of the Royal Greenwich Observatory. The agreement between these and the equivalent DE 102 data is close to 0.01 deg as far back as 601 BC, sound evidence in favour of the reliability of our tabular data.

Brief remarks are needed on the question of the clock error AT. This arises from a gradual increase in the length of the day due to a combination of lunar and solar tides and other causes. From an extensive study of historical observations, mainly of eclipses and occultations, Stephenson and Morrison (1984) deduced revised expressions for zXT. The difference between values calculated from these formulas and figures based on equation (1) above is less than one hour at all periods back to 601 BC, which is negligible for the present purpose. For consistency, in producing the present tables we have used equation (1) to calculate/xT values.

We have taken the opportunity to increase the versatility of the present volume by including- at 10-day intervals - the apparent magnitudes of each of the five bright planets Mercury, Venus, Mars, Jupiter and Saturn. We ourselves have often felt the need for readily accessible data of this kind. The values tabulated here should be especially useful in the case of Mercury and Mars, both of which fluctuate in brightness to a considerable degree. In not much more than a month, the magnitude of Mercury can vary from about - 1.5 at superior conjunction to fainter than + 3 at inferior conjunction. Changes in the brightness of Mars are much slower, but between opposition and conjunction the magnitude varies from about -2 to + 2. Both planets revolve in rather elliptical orbits. As a result, the actual range in magnitude for Mercury at superior conjunction is from -0.8 to -1.6 whereas for Mars at opposition the range is between -1.2 and -2.6. At a close opposition, Mars can thus briefly outshine Jupiter. The brightness of Venus, Jupiter and Saturn is relatively steady, seldom varying over a range of much more than about one magnitude. The main factor in the case of Saturn is the visibility of the ring system; due to the changes in the aspect of the rings, the opposition magnitude of this planet varies between about +0.7 and -0.3. In the tables we have computed the apparent magnitudes of the planets from the formulae derived by Mtiller (1893). These formulae, which are based on numerous observations, formed the basis of the data in the Astronomical Almanac until 1983. The differences between these and the newer formulae employed- due to Harris (1961)- are trivial for all practical purposes.

USE OF TABLES

The accompanying pages of tables normally carry four years of data, the only exception being for the first ' page, covering -600 and -599 (i.e. 601 and 600 BC). This format has been chosen for convenience so that a typical single page of the present volume will correspond to an open double page of Tuckerman. Our tables actually extend to A. D. 1651, rather than 1649. Column by column for each year we have: (i) the ecliptical longitude of Mars; (ii) the latitude of the planet; (iii) the apparent magnitude of Mars; (iv) the longitude of the Sun- given for reference; (v) the Julian Calendar date; (vi) to (ix) the apparent magnitudes of Mercury, Venus, Jupiter and Saturn. Interpolation of the magnitude data should be accurate except in the rare instances for Mercury and Venus when these planets pass very close to the Sun at inferior conjunction-a transit across the solar disc being an extreme example. The planetary symbols appearing at the top of the page are:

O	Mars	U	Venus
Q	Sun	V	Jupiter
S	Mercury	W	Saturn

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Sample: Mars, solar longitudes and magnitudes; January 9, -567 to 28 July, -567.

LONG	U LAT	MAG	Q LONG	**16.00 UT -567**	S M	T M	V M	W M
10.71	0.76	0.8	284.18	9JA	-0.2	-3.4	-1.5	1.3
16.08	0.88	1.0	294.26	19JA	-0.4	-3.4	-1.5	1.3
21.61	0.97	1.2	304.30	29JA	-0.9	-3.3	-1.6	1.3
27.27	1.05	1.3	314.28	8FE	-1.4	-3.3	-1.7	1.3
33.02	1.12	1.4	324.20	18FE	-1.1	-3.3	-1.7	1.3
38.85	1.17	1.5	334.06	28FE	-0.1	-3.3	-1.8	1.3
44.75	1.21	1.6	343.86	10MR	1.5	-3.3	-1.9	1.3
50.69	1.24	1.7	353.61	20MR	3.4	-3.4	-2.0	1.2
56.68	1.26	1.8	3.30	30MR	2.4	-3.4	-2.0	1.2
62.70	1.28	1.8	12.94	9AP	1.4	-3.4	-2.1	1.2
68.75	1.29	1.9	22.54	19AP	0.8	-3.4	-2.1	1.2
74.84	1.29	1.9	32.11	29AP	0.1	-3.5	-2.2	1.2
80.96	1.29	2.0	41.65	9MY	-0.8	-3.5	-2.2	1.1
87.10	1.28	2.0	51.17	19MY	-1.7	-3.6	-2.2	1.1
93.28	1.27	2.0	60.69	29MY	-1.2	-3.6	-2.1	1.1
99.50	1.26	2.0	70.21	8JN	-0.4	-3.7	-2.1	1.0
105.75	1.24	2.0	79.74	18JN	0.2	-3.8	-2.0	1.0
112.03	1.21	2.0	89.29	28JN	0.6	-3.9	-2.0	0.9
118.36	1.18	2.0	98.86	8JL	1.0	-4.0	-1.9	0.9
124.74	1.15	2.0	108.48	18JL	1.8	-4.1	-1.9	0.8
131.17	1.11	2.0	118.14	28JL	3.2	-4.2	-1.8	0.8

(Houlden and Stephenson 1986)